Microstructure Mechanics

Dislocation statics

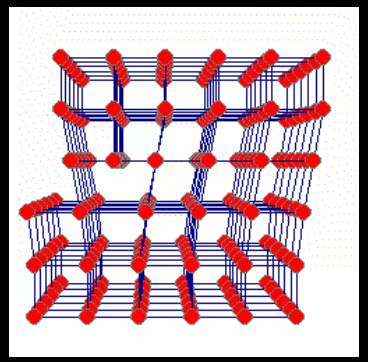
Dierk Raabe

Max-Planck-Institut
für Eisenforschung GmbH
Düsseldorf

Class 2011

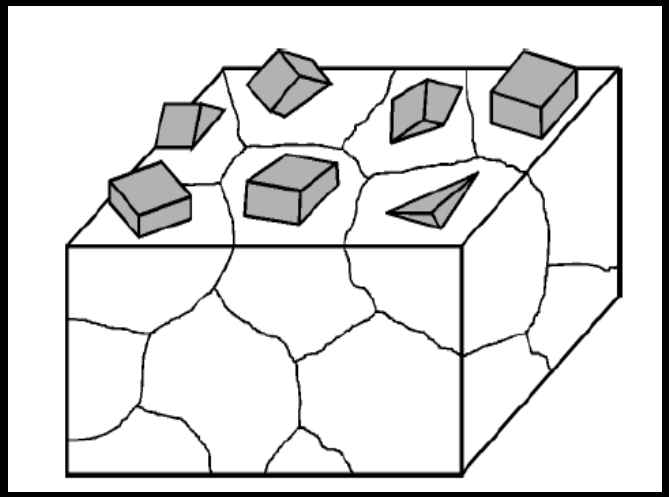
Hierarchy of dislocation mechanics

parallel loops (Kubin)

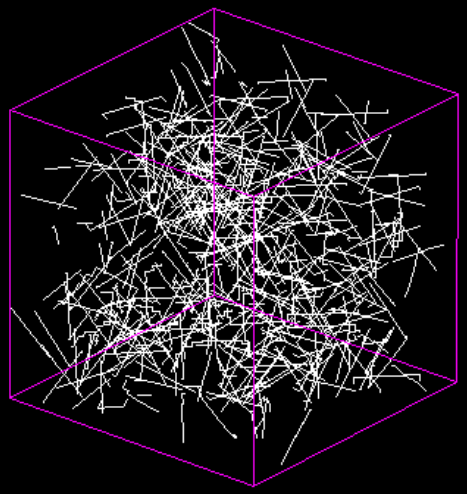


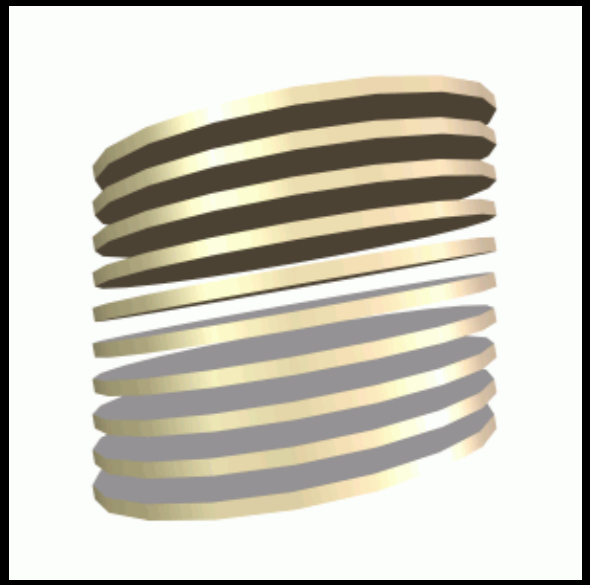
one dislocation

mesoscopic boundary conditions
(grain / orientation neighborhood)



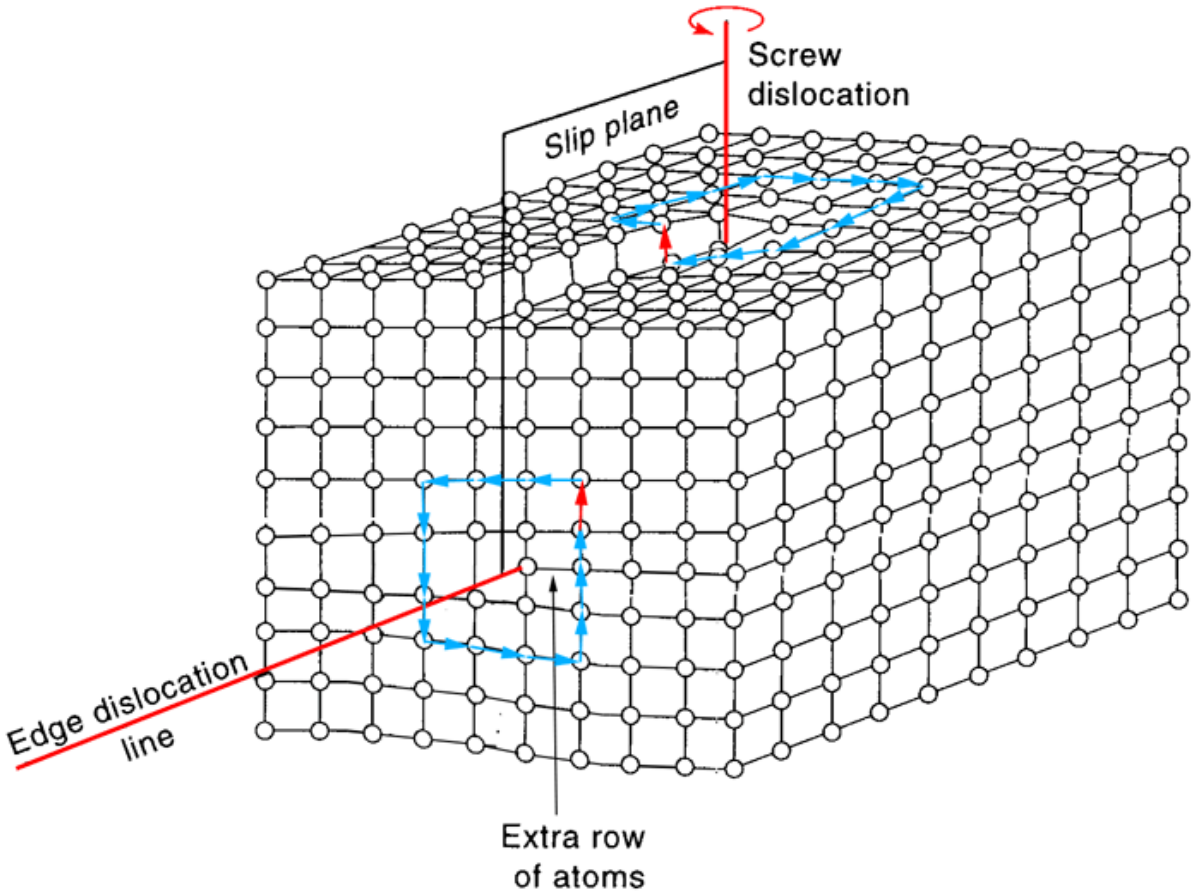
reactions (Kubin)





spin (orientation change)

Dislocation character



Kinematics, displacement

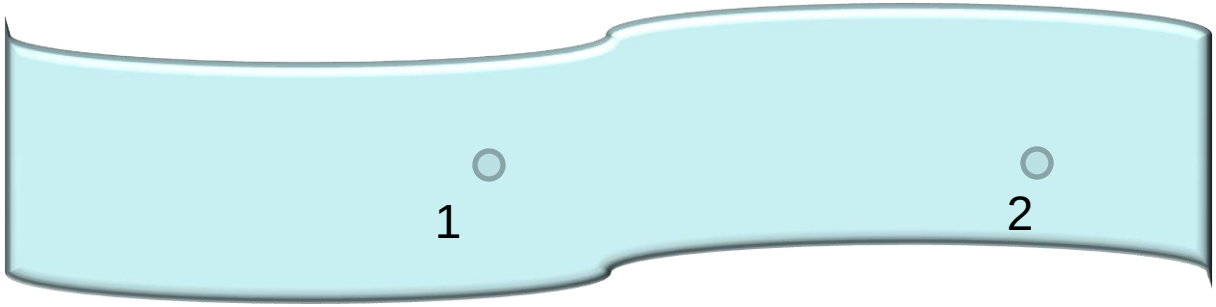
$\underline{u}=u(x,y,z)$

$\underline{u}_{(1)}(x,y,z)=\underline{u}_{(2)}(x,y,z)$

Kinematics, displacement

$\underline{u}=u(x,y,z)$

$\underline{u}_{(1)}(x,y,z)\neq\underline{u}_{(2)}(x,y,z)$



Kinematics, displacement

Displacement vector:

$$\mathbf{u} = [u_x, u_y, u_z]$$

Strain tensor:

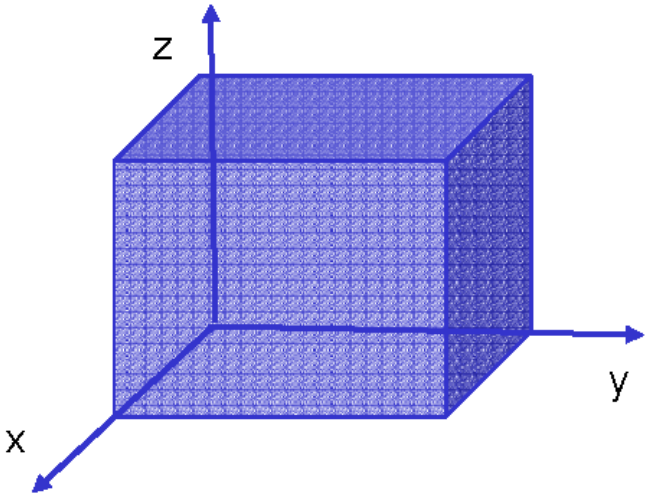
$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

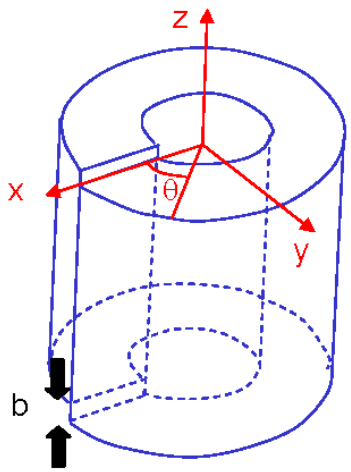
etc.

Dilatation, Δ:

$$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$



Strain field of straight screw dislocation



"Recipe" :
- take a hollow cylinder, axis along z:
- cut on a plane parallel to the z-axis;
-displace the free surfaces by **b** in the z-direction.

By inspection:

$$u_x = u_y = 0$$

$$u_z = \frac{b\theta}{2\pi} = \frac{b}{2\pi}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{yx} = 0$$

$$\varepsilon_{xz} = \tfrac{1}{2}\frac{\partial u_z}{\partial x} = \frac{b}{4\pi}\frac{\partial}{\partial x}\tan^{-1}\left(\frac{y}{x}\right) = -\frac{b}{4\pi}\frac{1}{1+\left(y/x\right)^2}\frac{y}{x^2} = -\frac{b}{4\pi}\frac{y}{x^2+y^2} = -\frac{b}{4\pi}\frac{\sin\theta}{r}$$

$$\varepsilon_{yz} = \tfrac{1}{2}\frac{\partial u_z}{\partial y} = \frac{b}{4\pi}\frac{\partial}{\partial y}\tan^{-1}\left(\frac{y}{x}\right) = \frac{b}{4\pi}\frac{1}{1+\left(y/x\right)^2}\frac{1}{x} = \frac{b}{4\pi}\frac{x}{x^2+y^2} = \frac{b}{4\pi}\frac{\cos\theta}{r}$$

Stress field of straight screw dislocation

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{yx} = 0$$

$$\varepsilon_{xz} = -\frac{b}{4\pi}\frac{y}{x^2+y^2} = -\frac{b}{4\pi}\frac{\sin\theta}{r}$$

$$\varepsilon_{yz} = \frac{b}{4\pi}\frac{x}{x^2+y^2} = \frac{b}{4\pi}\frac{\cos\theta}{r}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0$$

$$\Delta = 0$$

$$\sigma_{xz} = 2G\varepsilon_{xz} = -\frac{Gb}{2\pi}\frac{y}{x^2+y^2} = -\frac{Gb}{2\pi}\frac{\sin\theta}{r}$$

$$\sigma_{yz} = 2G\varepsilon_{yz} = \frac{Gb}{2\pi}\frac{x}{x^2+y^2} = \frac{Gb}{2\pi}\frac{\cos\theta}{r}$$

In Polar coordinates:

(either by direct inspection, or by transforming the strains and stresses from Cartesian co-ordinates)

$$\varepsilon_{\theta z} = \varepsilon_{z\theta} = \frac{b}{4\pi r}$$

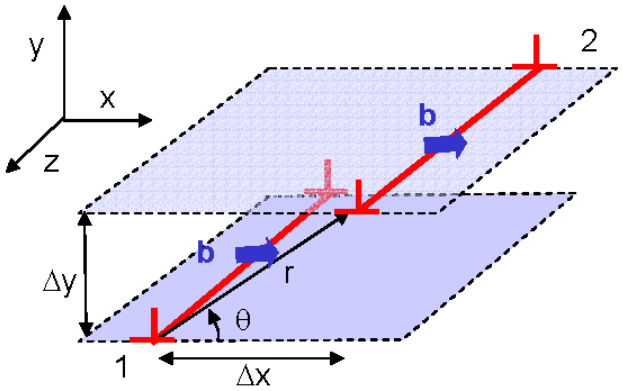
$$\sigma_{\theta z} = \sigma_{z\theta} = \frac{Gb}{2\pi r}$$

All other components of the stress tensor are zero.

Note:
- Stress and strain fields are pure shear
- Fields have radial symmetry
- Stresses and strains are proportional to 1/r:
  - extend to infinity
  - tend to infinite values as $r \Rightarrow 0$

Infinite stresses cannot exist in real materials: the dislocation core radius r_0 is that within which our assumption of linear elastic behaviour breaks down. Typically $r_0 \approx 1$ nm.

Forces between edge dislocations



Dislocation 2 "feels" the stress field of dislocation 1 (and vice versa).

The important components of the stress field are:

$\boldsymbol{\sigma_{xy}}$ – produces *glide* force on disln 2;

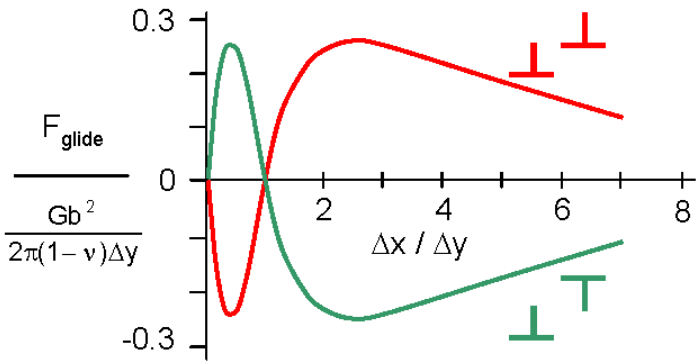
$\boldsymbol{\sigma_{xx}}$ – produces *climb* force on disln 2.

$$\sigma_{xx} = -D\,y\,\frac{3\Delta x^2 + \Delta y^2}{\left(\Delta x^2 + \Delta y^2\right)^2}, \quad \text{with}: \; D = \frac{Gb}{2\pi(1-\nu)}$$

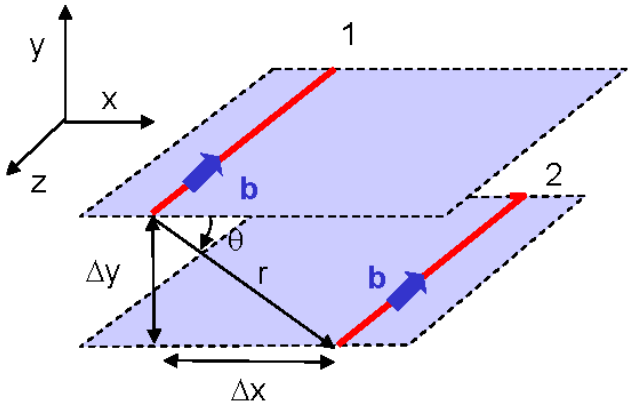
$$\sigma_{xy} = \sigma_{yx} = D\,\Delta x\,\frac{\Delta x^2 - \Delta y^2}{\left(\Delta x^2 + \Delta y^2\right)^2}$$

So glide force, resolved onto the slip plane, is:

$$F_{glide} = \frac{Gb^2}{2\pi(1-\nu)}\,\frac{\Delta x(\Delta x^2 - \Delta y^2)}{\left(\Delta x^2 + \Delta y^2\right)^2}$$



Forces between screw dislocations



Dislocation 2 "feels" the stress field of dislocation 1 (and vice versa).

$$\sigma_{\theta z} = \sigma_{z\theta} = \frac{Gb}{2\pi r}$$

So force on dislocation 2 from dislocation 1 is:

$$F = \frac{Gb^2}{2\pi r}$$

… but this force acts in the radial direction.

Force on dislocation 2 from dislocation 1, resolved onto the glide plane is:

$$F_{res} = \frac{Gb^2}{2\pi r}\cos\theta$$

Alternatively, we can use the stress field expressed in Cartesian co-ordinates:

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0$$

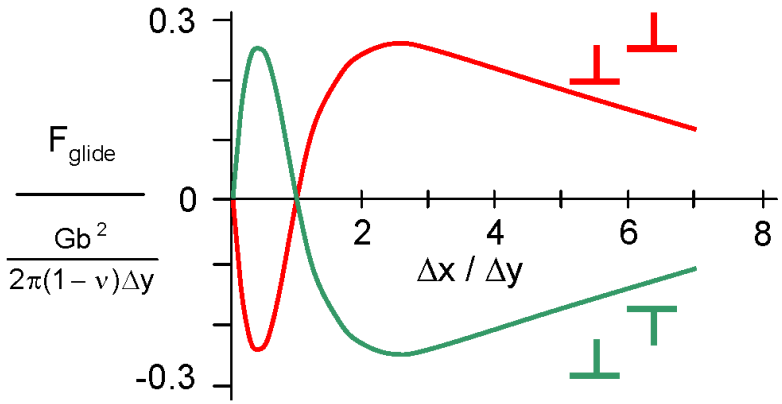
$$\sigma_{xz} = -\frac{Gb}{2\pi}\frac{\Delta y}{\Delta x^2 + \Delta y^2} = -\frac{Gb}{2\pi}\frac{\sin\theta}{r}$$

$$\sigma_{yz} = \frac{Gb}{2\pi}\frac{\Delta x}{\Delta x^2 + \Delta y^2} = \frac{Gb}{2\pi}\frac{\cos\theta}{r}$$

Note that the shear stress acting to shear atoms paralle to **b** above and below the glide plane is σ_{yz}.

$$F_{res} = \sigma_{yz} b = \frac{Gb^2}{2\pi r}\cos\theta = \frac{Gb^2}{2\pi}\frac{\Delta x}{\Delta x^2 + \Delta y^2}$$

Stable arrangements for edge dislocations



For **like** Burgers vectors:
$\Delta x = \pm\Delta y$: unstable equilibrium
$\Delta x = 0$: stable equilibrium

For **opposite** Burgers vectors:
$\Delta x = \pm\Delta y$: stable equilibrium
$\Delta x = 0$: unstable equilibrium

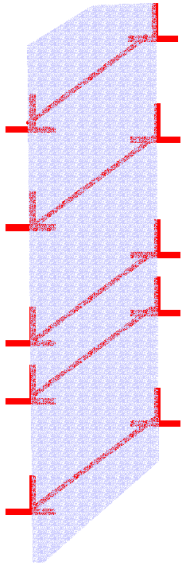
For a set of **"opposite"** Burgers vectors:

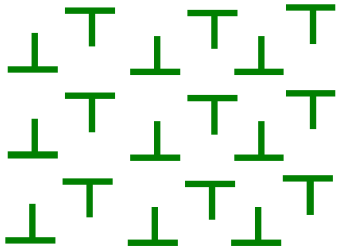
There are a large number of possible stable arrangements.

For **like** Burgers vectors:
Stable array is a planar stack

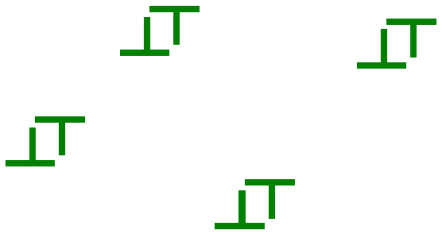
A low angle tilt boundary.

This arrangement has a strong long-range stress field.





"Taylor lattice"



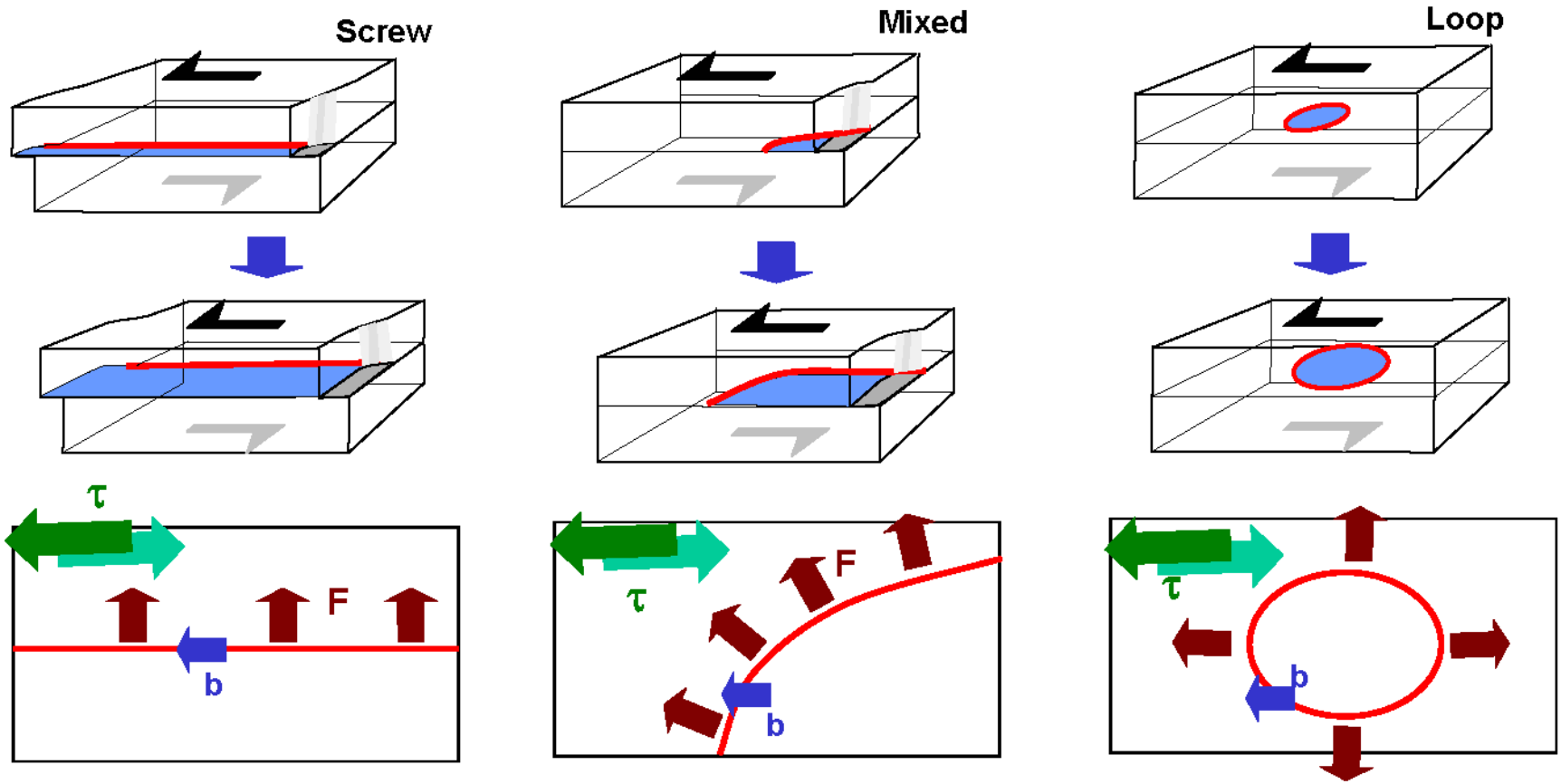
"Dipole dispersion"

These stable arrangements have minimal *long-range* stress fields.

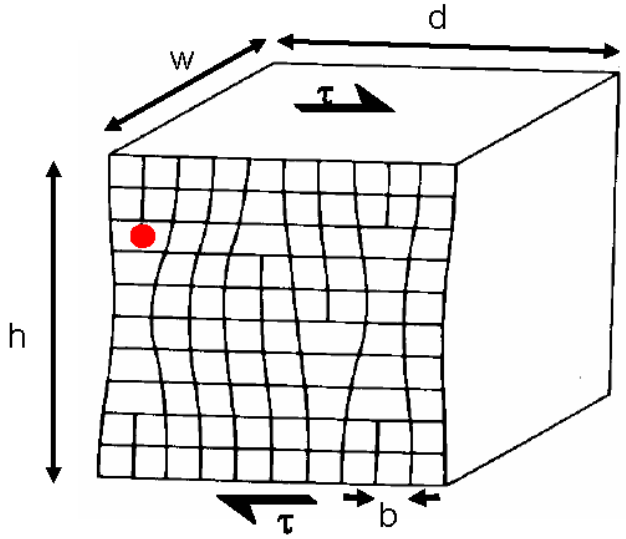
Forces on dislocations

Dislocation motion only has "meaning" normal to the line vector.

Forces on dislocations can only act normal to the line vector.



Strain rate from motion of dislocations



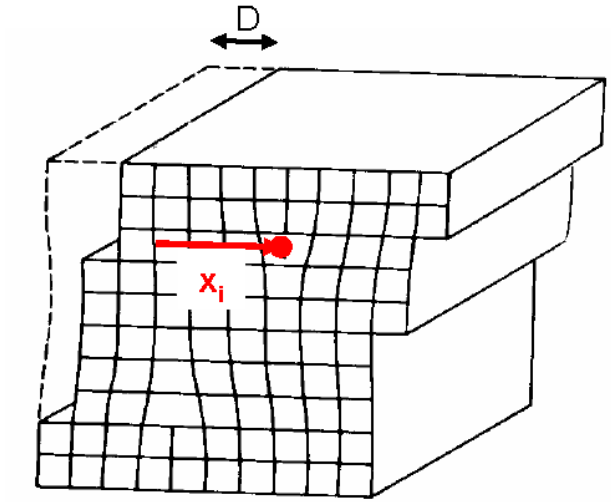
If a dislocation moved the whole length of the crystal d, it would contribute **b** to the displacement **D.**

If each dislocation moves an amount x_i (less than d), ther each will contribute (x_i / d).**b** to **D**.

$$D = \frac{b}{d}\sum_i x_i$$

Shear strain is:

$$\varepsilon = \frac{D}{h} = \frac{b}{dh}\sum_i x_i$$



Define average distance moved by each dislocation:

$$\bar{x} = \frac{1}{N}\sum_i x_i$$

Density of *mobile* dislocations is:

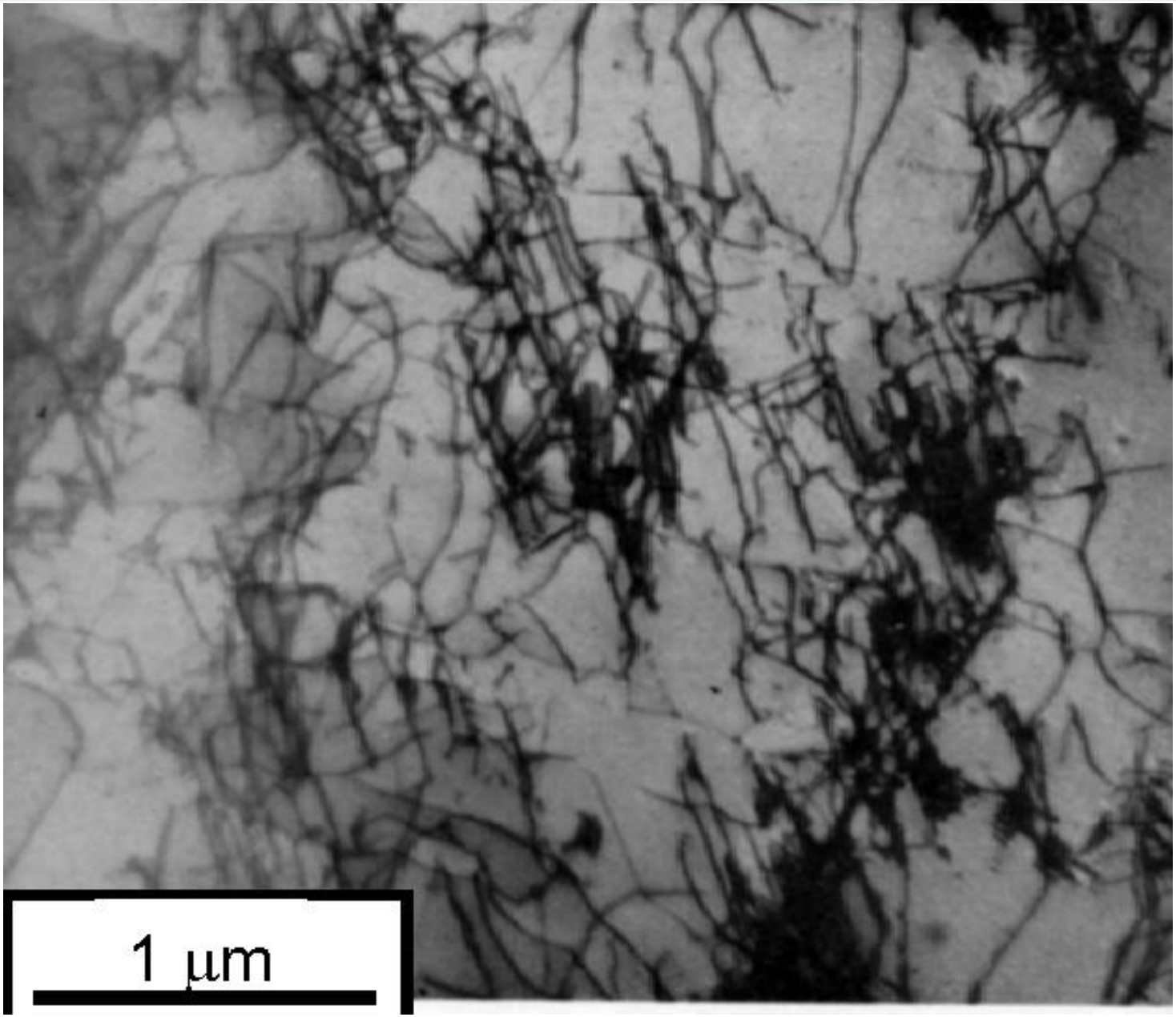
$$\rho_m = \frac{N}{hd}$$

Strain rate:

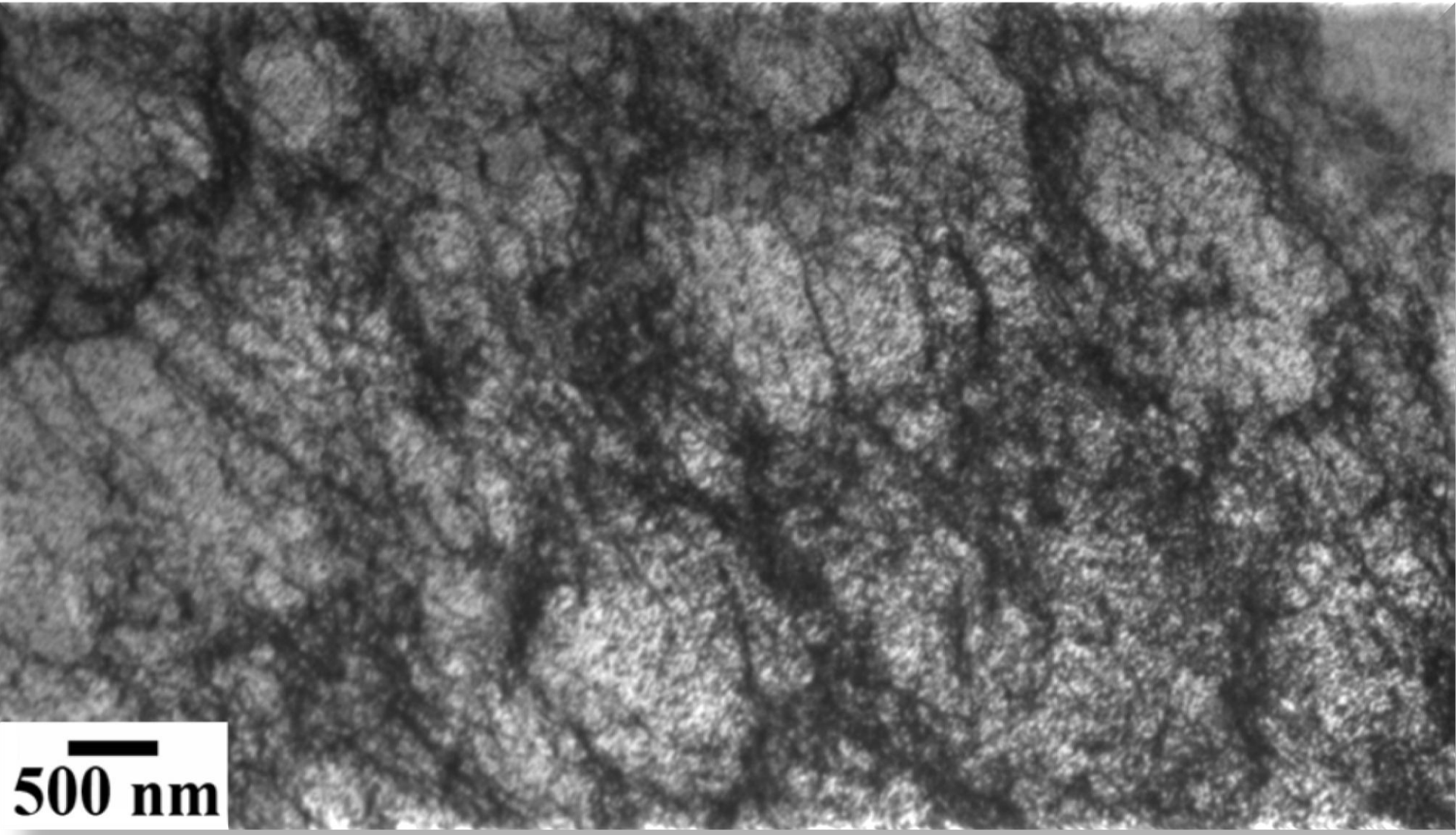
$$\boxed{\dot{\varepsilon} =} \frac{d\varepsilon}{dt} = b\rho_m \frac{d\bar{x}}{dt} = \boxed{b\rho_m \bar{v}}$$

- where $\bar{v}$ is the average dislocation glide velocity.

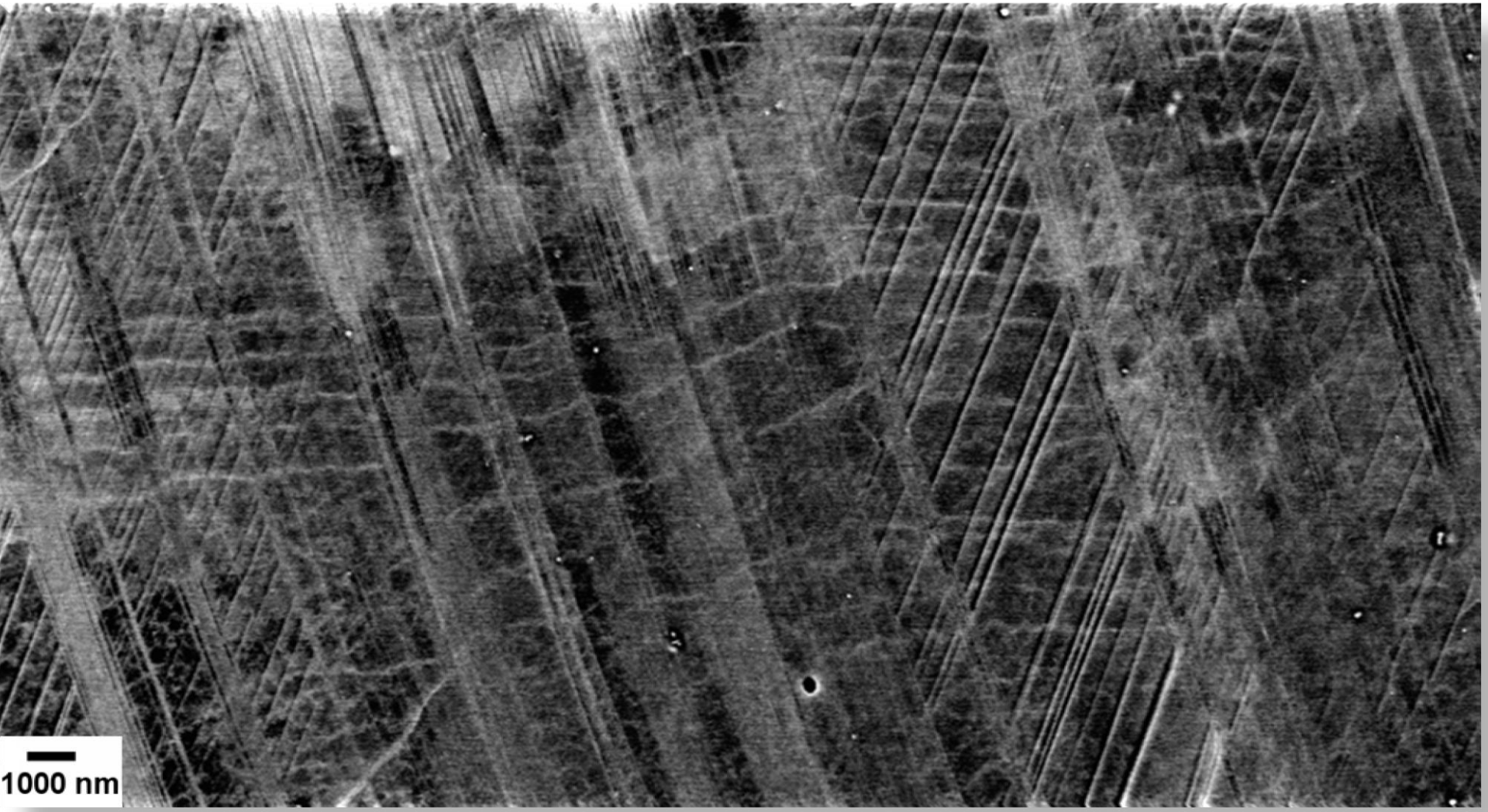
Real dislocations



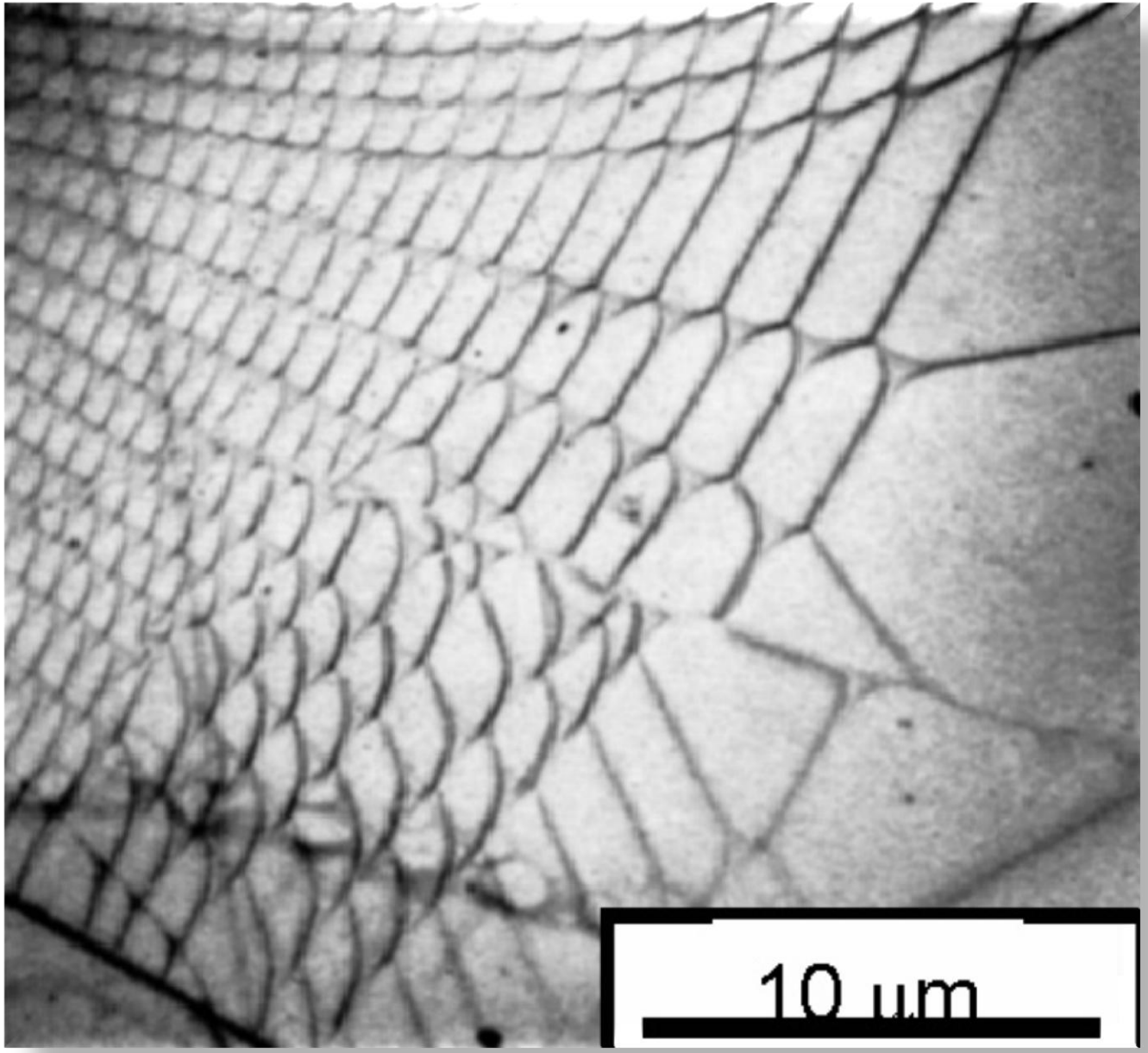
Real dislocations



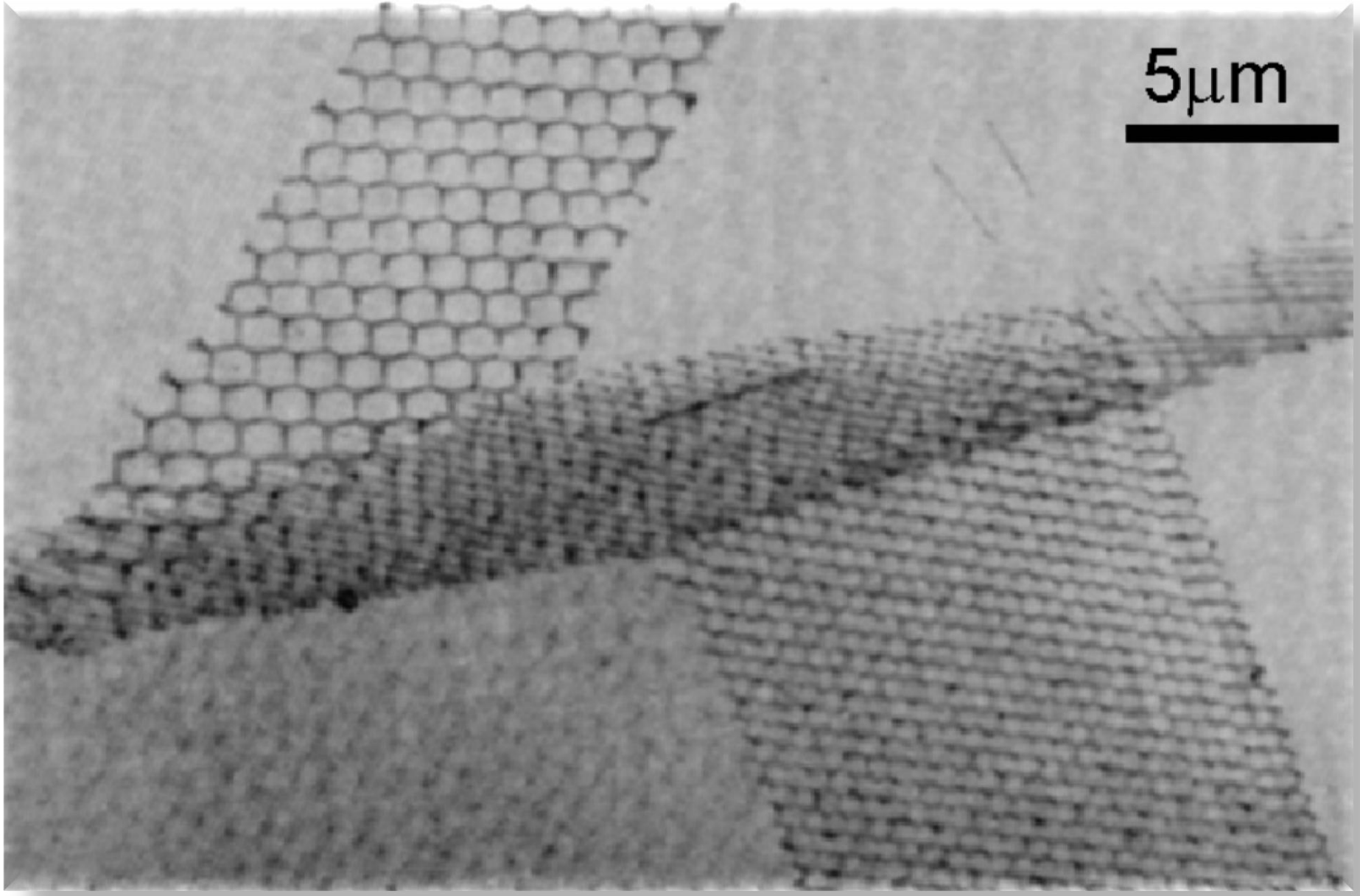
Real dislocations



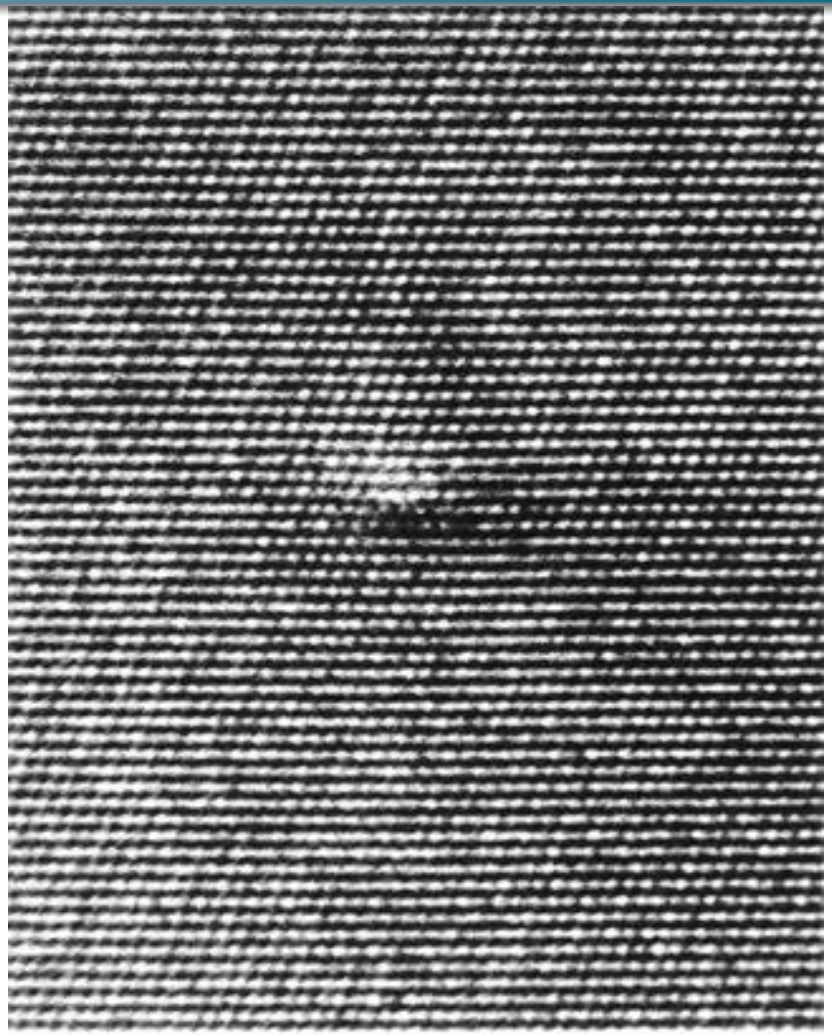
Real dislocations



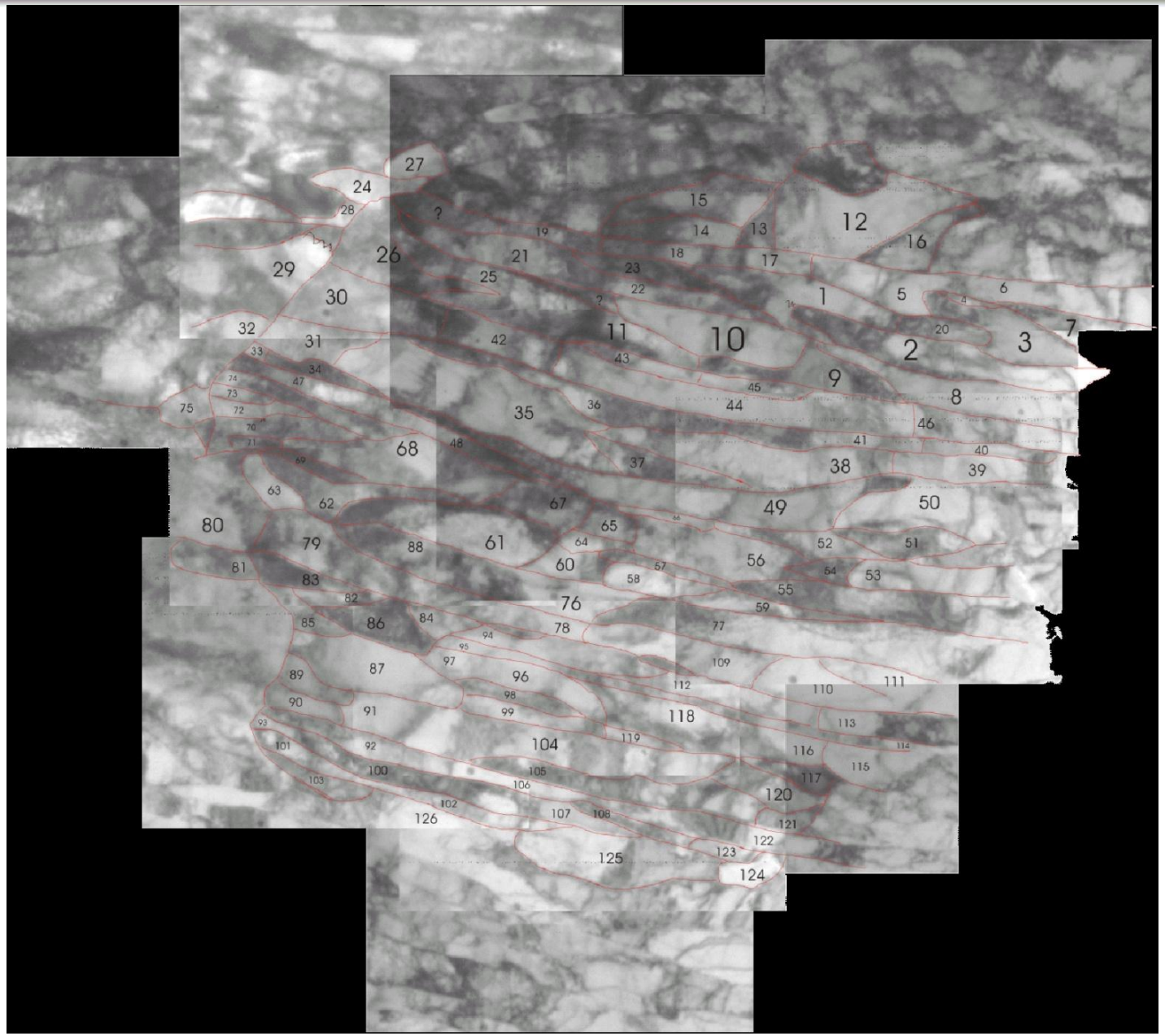
Real dislocations



Real dislocations



Real dislocations



Real dislocations